Structure of the σ-meson and diamagnetism of the nucleon

Probing the nucleon structure by two-photon reactions

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- Polarizabilities and nucleon structure

- The sigma meson degree of freedom

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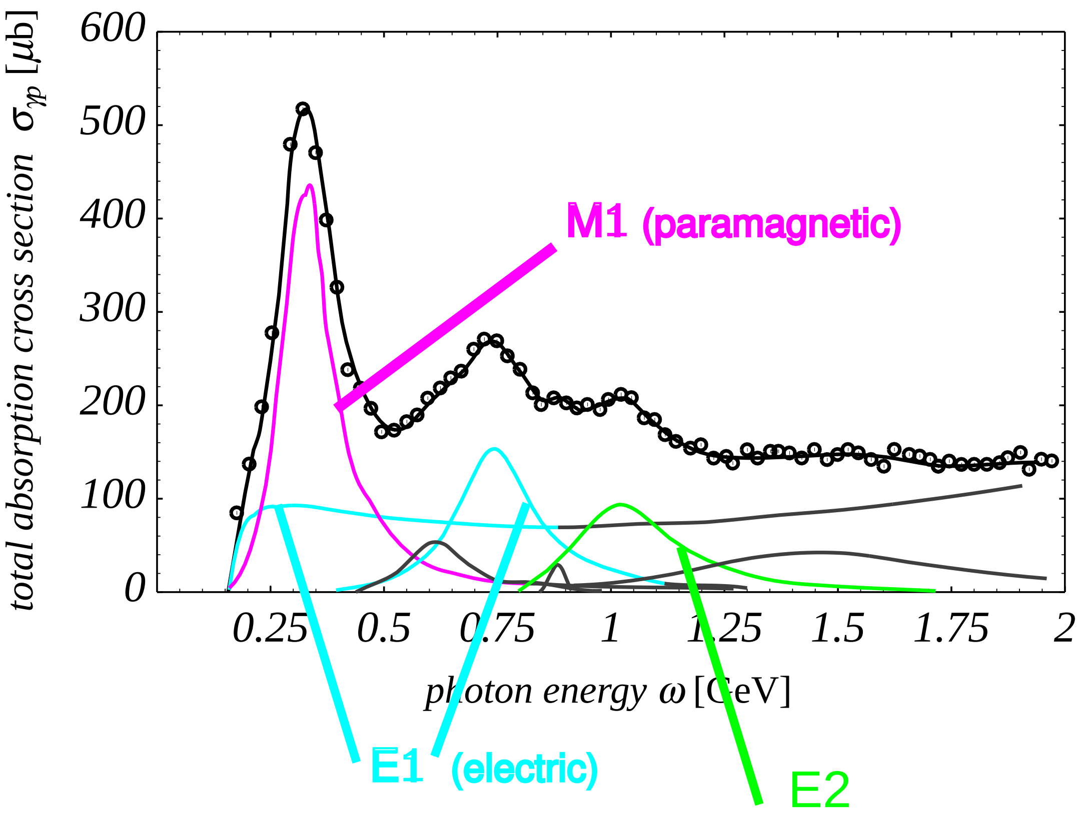
Multipole composition of electromagnetic polarizabilities



Baldin's sum rule: $\alpha + \beta = \frac{1}{2\pi^2} \int_{\omega_{thr}}^{\infty} \frac{\sigma_{tot}(\omega)}{\omega^2} d\omega$

$\alpha_p + \beta_p = (13.9 \pm 0.3) \times 10^{-4}\,\mathrm{fm}^3$ for the proton

$\alpha_n + \beta_n = (15.2 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3$ for the neutron

M. Schumacher, Prog.Part. Nucl. Phys. 55 (2005) 567 [hep-ph/0501167]

Estimate of polarizabilities:

$\alpha_p \sim 4 \times 10^{-4}\,\mathrm{fm}^3$

$\alpha_n \sim 5 \times 10^{-4}\,\mathrm{fm}^3$

$\beta_p \sim 7 \times 10^{-4}\,\mathrm{fm}^3$

$\beta_n \sim 7 \times 10^{-4}\,\mathrm{fm}^3$

Experimental results:

$\alpha_p = (12.0 \pm 0.6) \times 10^{-4}\,\mathrm{fm}^3$

$\alpha_n = (12.5 \pm 1.7) \times 10^{-4}\,\mathrm{fm}^3$

$\beta_p = (1.9 \mp 0.6) \times 10^{-4}\,\mathrm{fm}^3$

$\beta_n = (2.7 \mp 1.8) \times 10^{-4}\,\mathrm{fm}^3$

Compton scattering and the σ-meson

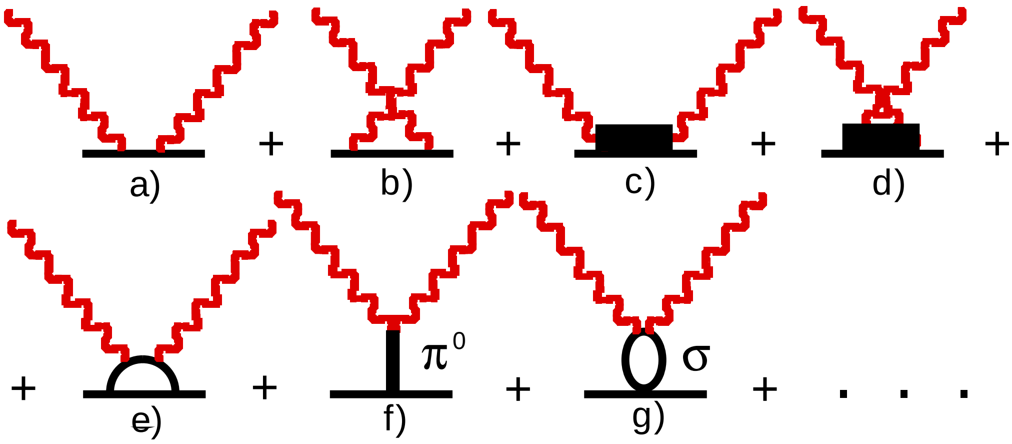


Fig. 1: Diagrams of Compton scattering

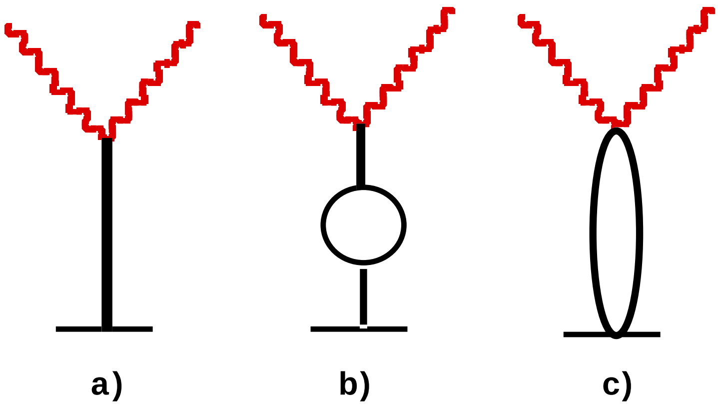


Fig. 2: Diagrams of the σ-meson

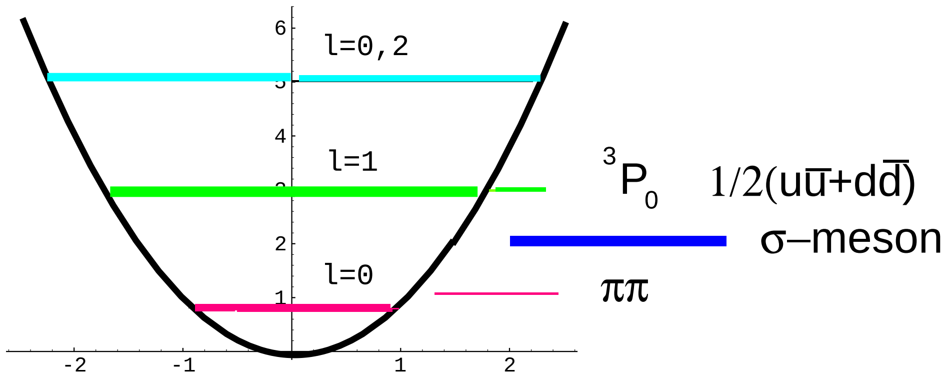
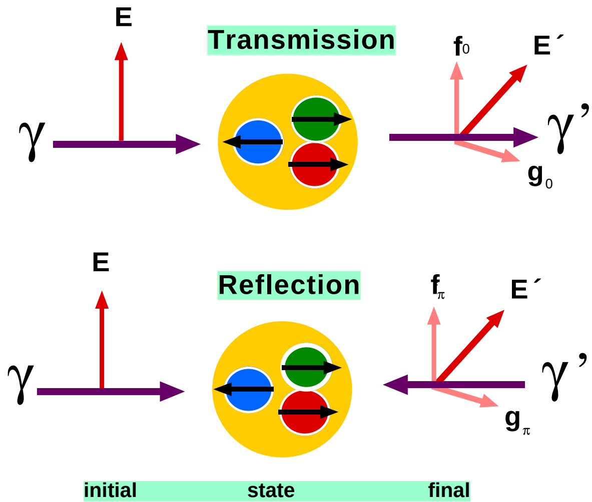


Fig. 3: Structure of the σ-meson

Forward and backward Compton scattering



$$T^{\mathrm{LAB}}(\theta = 0) \quad = \quad f_0(\omega)\, \epsilon' \cdot \epsilon + g_0(\omega)\, i\, \sigma \cdot (\epsilon' \times \epsilon)$$

$$T^{\mathrm{LAB}}(\theta = 0) \quad = \quad f_\pi(\omega)\, \epsilon' \cdot \epsilon + g_\pi(\omega)\, i\, \sigma \cdot (\epsilon' \times \epsilon)$$

$$f_0(\omega) \quad = \quad -\frac{e^2}{4\pi M} q^2 + \omega^2(\alpha + \beta) + \mathcal{O}(\omega^4) \ \text{Baldin}$$

$$g_0(\omega) \quad = \quad \omega \left[-\frac{e^2}{8\pi M^2} \kappa^2 + \mathcal{O}(\omega^2) \right] \ \text{GDH}$$

$$f_\pi(\omega) \quad \sim \quad -\frac{e^2}{4\pi M} q^2 + \omega\omega'(\alpha - \beta) + \mathcal{O}(\omega^2 \omega'^2) \ \text{BEFT}$$

$$g_\pi(\omega) \quad \sim \quad \frac{e^2}{8\pi M^2}(\kappa^2 + 4q\kappa + 2q^2) + (\omega\omega')\gamma_\pi + \mathcal{O}(\omega^2\omega'^2)\text{LN}$$

$$q \quad = \quad \frac{1}{2}(1 + \tau_3)$$

s-channel and t-channel parts of polarizabilities

$$(\alpha+\beta)^s \equiv \alpha+\beta = \frac{1}{2\pi^2}\int_{\omega_{\rm thresh}}^{\infty} \frac{\sigma_{\rm tot}(\omega)}{\omega^2} d\omega \quad \text{(Baldin)}$$

s-channel part of BEFT$^{(*)}$ sum rule:

$$(\alpha-\beta)^s = \frac{1}{2\pi^2}\int_{\omega_{\rm thresh}}^{\infty} \sqrt{1+\frac{2\omega}{M}}(\sigma_1(\omega)-\sigma_2(\omega))\frac{d\omega}{\omega^2}$$

$$\sigma_1 = \sigma_{(E1,M2,E3,\cdots)} \quad \text{parity change}$$

$$\sigma_2 = \sigma_{(M1,E2,M3,\cdots)} \quad \text{parity non-change}$$

proton	neutron	
$(\alpha+\beta)^s_p = 13.9 \pm 0.3$	$(\alpha+\beta)^s_n = 15.2 \pm 0.5$	Baldin
$(\alpha-\beta)^{s+t}_p = 10.1 \pm 0.9$	$(\alpha-\beta)^{s+t}_n = 9.8 \pm 2.5$	$\gamma\gamma$-exp.
$(\alpha-\beta)^s_p = -5.0 \pm 1.0$	$(\alpha-\beta)^s_n = -5.0 \pm 1.0$	BEFT
$(\alpha-\beta)^t_p = 15.1 \pm 1.3$	$(\alpha-\beta)^t_n = 14.8 \pm 2.7$	σ-meson
$\alpha^{s+t}_p = 12.0 \pm 0.6$	$\alpha^{s+t}_n = 12.5 \pm 1.7$	exp.
$\alpha^s_p = 4.5 \pm 0.5$	$\alpha^s_n = 5.1 \pm 0.6$	calc.
$\alpha^t_p = 7.5 \pm 0.8$	$\alpha^t_n = 7.4 \pm 1.8$	σ-meson
$\beta^{s+t}_p = 1.9 \mp 0.6$	$\beta^{s+t}_n = 2.7 \mp 1.8$	exp.
$\beta^s_p = 9.5 \pm 0.5$	$\beta^s_n = 10.1 \pm 0.6$	calc.
$\beta^t_p = -7.6 \pm 0.8$	$\beta^t_n = -7.4 \pm 1.9$	σ-meson

(*) J. Bernabeu, T.E.O. Ericson, C. Ferro Fontan, Phys. Lett. 49 B (1974) 381; J. Bernabeu, B. Tarrach, Phys. Lett. 69 B (1977) 484

Singularities in the s-channel and t-channel

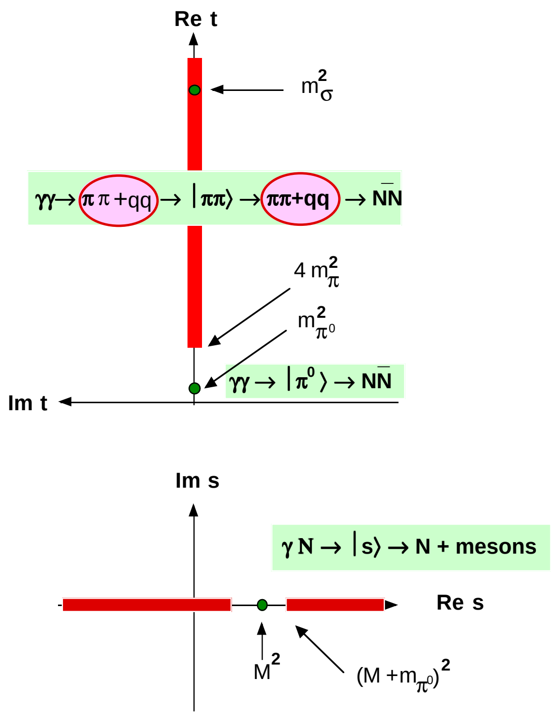


Fig. 1: Singularities in the s-channel and the t-channel

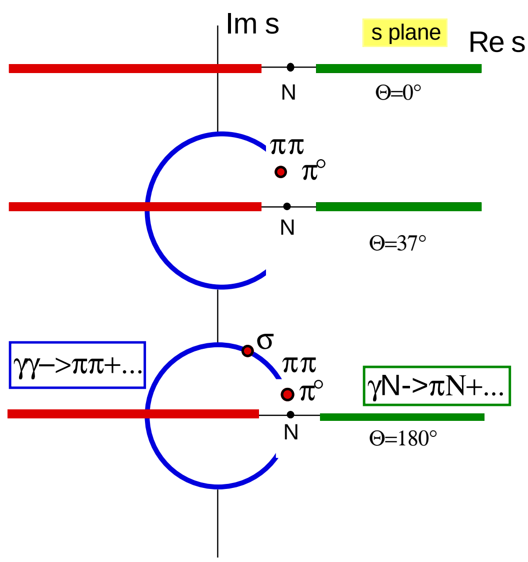


Fig. 2: t-channel singularities projected into the s-plane

A.C. Hearn, E. Leader, Phys. Rev. 126 (1962) 789

Predictions for $(\alpha-\beta)^t$

The t-channel part of the BEFT sum rule is given by

$$(\alpha-\beta)^t = \frac{1}{16\pi^2}\int_{4m_\pi^2}^{\infty}\frac{dt}{t^2}\frac{16}{4M^2-t}\left(\frac{t-4m_\pi^2}{t}\right)^{1/2}$$
$$\times\left[f_+^0(t)F_0^{0*}(t)-\left(M^2-\frac{t}{4}\right)\left(\frac{t}{4}-m_\pi^2\right)f_+^2(t)F_0^{2*}(t)\right]$$

$f_+^{J=0,2}(t)$ partial wave amplitude of the process $N\bar{N}\to\pi\pi$
$F_{I=0}^{J=0,2}(t)$ partial wave amplitude of the process $\pi\pi\to\gamma\gamma$

BEFT prediction: $(\alpha-\beta)^t = +(14.0\pm 2.0)$ Levchuk et al.
BEFT prediction: $(\alpha-\beta)^t = +16.5$ Dechsel et al.
BEFT average: $(\alpha-\beta)^t = \underline{\underline{+15.3}}$

Quark model calculation:

$$(\alpha-\beta)^t = \frac{5(e^2/4\pi)g_{\pi NN}}{6\pi^2 m_\sigma^2 f_\pi} = \underline{\underline{15.3}}$$

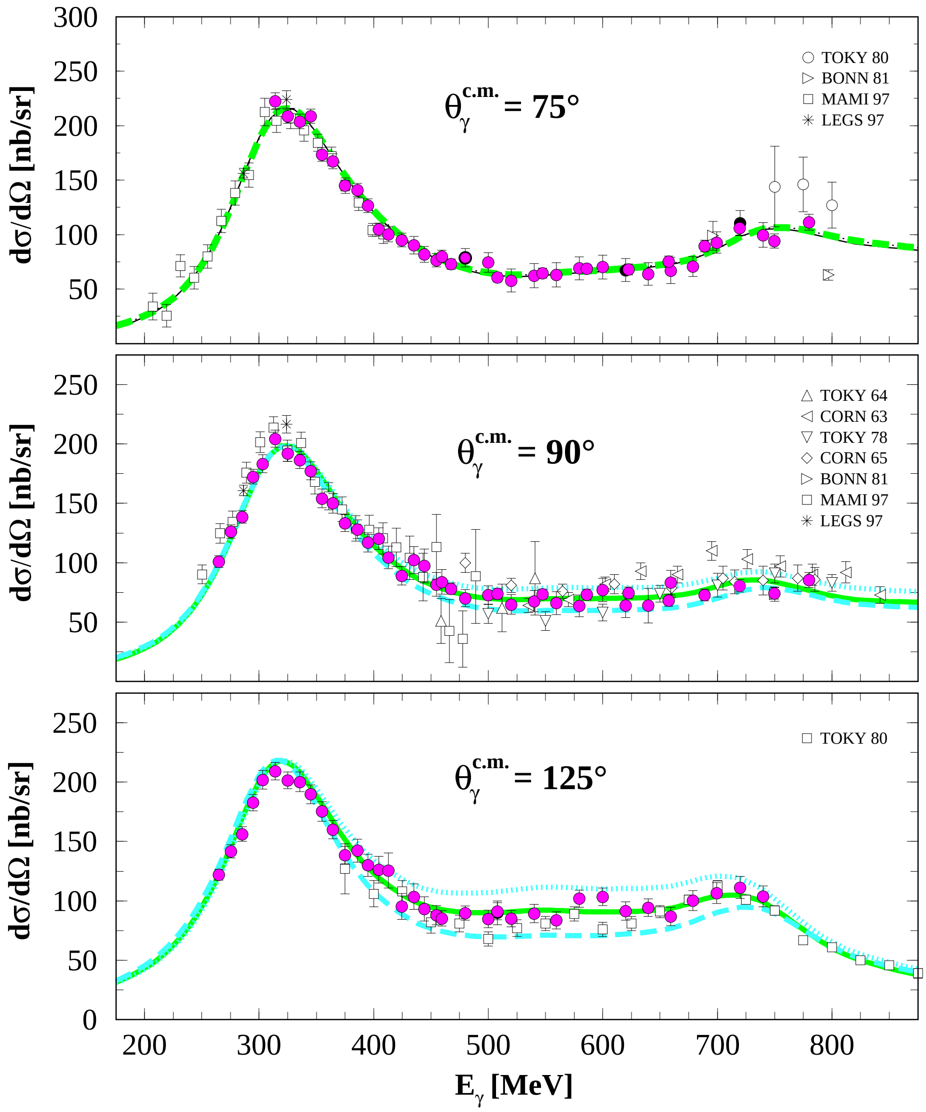
with $g_{\pi NN}=13.17$, $m_\sigma = 665 MeV$, $f_\pi = 92.43 MeV$

Experimental results:

$$(\alpha-\beta)_p^t = \underline{\underline{15.1\pm 1.3}}$$

$$(\alpha-\beta)_n^t = 14.8\pm 2.7$$

Experimental evidence for the effective σ-pole



$m_{\text{eff}} = 800$ MeV (upper curve), $m_{\text{eff}} = 600$ MeV (central curve), $m_{\text{eff}} = 400$ MeV (lower curve)
Propagator of the σ-meson

$$\frac{1}{t - M_\sigma^2 - \Sigma(t)} = \frac{1}{t - m_\sigma^2 - \mathcal{P}(t)} \qquad (1)$$

with M_σ = bare σ mass, $\Sigma(t)$ = self-energy, m_σ = experimental σ mass, $\mathcal{P}(t) = \Sigma(t) - \Sigma(o)$ = renormalized self-energy, $m_{\text{eff}}^2 = m_\sigma^2 + \mathcal{P}(t)$

Summary

- It has been shown that there is a very close relation between the structure of the σ meson and the diamagnetism of the nucleon

- The σ meson is composed of a $q\bar{q}$ configuration in a confining potential and a $\pi\pi$ state in the continuum. These two parts are strongly coupled, thus forming a particle with a mass of 665 MeV.

- The couplings of the σ meson to two photons and to the constituent quarks in a nucleon proceed through the $q\bar{q}$ structure component. This is the reason that the contribution of the σ meson degree of freedom to the difference $(\alpha - \beta)$ of electromagnetic polarizabilities can be calculated in two ways.

- The first calculation makes use of the $q\bar{q}$ structure of the σ meson, whereas the second calculation exploites the properties of the σ meson as a correlated π meson pair. Both calculations lead to an agreement with each other and to an agreement with experiment.